

values and data at low and varying L_H/D_T (Dow and Jakob). Furthermore the Van Heerden study was the only one in the external surface study that had only part of the bed in contact with the heat transfer surface, and there was thus an apparatus geometry difference between that study and all the others used.

The Van Heerden data should be studied however, and we have been gradually working them up since the publication of our paper. Our calculations confirm the deviations cited by Leva and Wen and point up the complexity of the

low L_H/D_T region in the case of external surface. Our paper points out that the external surface correlation is uncertain in the low L_H/D_T region, and it is hoped that further studies will enable us to improve it in this region.

As to the differences in slope between curves of Van Heerden data and the external surface correlation curve, mentioned by Leva and Wen, we believe these to be caused not by an improper incorporation of the functional relationship between fluid mass velocity and bed expansion but by the uncertainty of

the slope of the correlation curve in the low Reynolds number region resulting from the very small number of points in that region.

The differences in form between our external and internal surface correlations are indeed substantial but were found necessary in our empirical study. However it may still be hoped that more work in this field by others will yet produce a definitive correlation, theoretically based and covering in a single expression both external and internal surface operation.

Diffusion in a Pore of Irregular Cross Section—a Simplified Treatment

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In a recent issue of this journal Petersen* developed an expression for the convergence factor for diffusion through a pore whose cross section varied periodically along its length in a hyperbolic fashion, from which he demonstrated how the effective diffusivity in such a pore varied with the ratio of maximum to minimum cross-sectional area. This writer has had occasion to grapple with this same problem; by means of a very elementary analysis of the diffusion process and with the aid of some simplifying assumptions, he has developed a relation between the effective diffusivity and pore geometry that is in remarkably close agreement with Petersen's far more elegant analysis.

Consider two cylindrical capillaries, one of radius r_1 and length l_1 and the other of radius r_2 and length l_2 , which are connected in series. An irregular pore may be considered as composed of any number of these characteristic units in series. Since the diffusional resistance offered by any one of these units is the same as any other, it is possible by characterizing the unit to characterize thereby the entire pore.

A component is assumed to be diffusing through this unit at steady state. If entrance and exit effects at the juncture of the two capillaries are neglected, the flux N through the two capillaries can be represented by

$$N = D(\pi r_1^2) \left(\frac{c_i - c'}{l_1} \right) \quad (1)$$

$$= D(\pi r_2^2) \left(\frac{c' - c_o}{l_2} \right)$$

If one defines two dimensionless ratios $L = l_1/l_2$ and $R = r_1/r_2$, Equation (1) can be solved for c' and the flux expressed in terms of the inlet and outlet concentration:

$$N = D \left(\frac{\pi r_1^2}{l_1} \right) (c_i - c_o) \left[\frac{L}{R^2 + L} \right] \quad (2)$$

An equivalent uniform cylindrical pore can be defined as one with an average volume per unit length that is the same as that of the irregular pore*; that is,

$$\pi r_e^2 (l_1 + l_2) = \pi r_1^2 l_1 + \pi r_2^2 l_2^2 \quad (3)$$

The flux through the irregular pore can be equated to that through the equivalent pore, if an effective diffusivity is substituted for the diffusion coefficient.

$$N = D_e (\pi r_e^2) \frac{c_i - c_o}{l_1 + l_2} \quad (4)$$

Substituting Equation (3) into (4) and equating the resulting relation to Equation (2), one finds that

$$D/D_e = \left[1 + \frac{L}{(L+1)^2} \left(\frac{R^2 - 1}{R} \right)^2 \right] \quad (5)$$

Equation (5) permits calculation of the convergence factor D_e/D as a function of

R and L . The parameter R alone (or more correctly, R^2) appears in Petersen's derivation, since his assumed hyperbolic variation of pore cross section with length makes (L) and (R) interdependent.

In Figure 1 (D_e/D) is plotted as a function of R for various values of L . The dotted line on the graph represents Petersen's correlation. It will be noted that the two correlations are in rather close agreement for values of R up to about five, irrespective of the value of L . At higher values of R however they diverge appreciably. In view of the fact that with Petersen's hyperbolic construction of the pore the fraction of the pore length containing the bulges increases with the ratio of maximum to minimum pore diameter (that is, L increases with R), the observed divergence of the two correlations at high values of R is to be expected.

In the absence of specific microscopic information about pore geometry, it is convenient to assume some arbitrary relation between L and R in order to express the convergence factor in terms of a single geometric parameter. One not unreasonable assumption is that the length of a bulge or constriction is proportional to its diameter, that is, that $L = R$. In this case Equation (5) becomes

$$D_e/D = \left[1 + \frac{(R-1)^2}{R} \right]^{-1} \quad (6)$$

Equation (6) yields values of the convergence factor which are about 30% lower (at higher values of R) than Petersen's. In view of the simplifying assumptions employed in the present development this agreement is surprising. If Equation (5) is solved for the case

*Alternatively an equivalent cylindrical pore can be defined as one with a surface area per unit volume equal to that of the real pore; in this case the volume of the equivalent pore per unit length will be less than that of the real pore, and a number of equivalent pores in parallel must be selected such that the total pore volume per unit length is the same as that of the real pore. Since the diffusional flux through a uniform cylindrical pore is directly proportional to its cross-sectional area, these two definitions of the equivalent pore yield the same algebraic solution.

*Petersen, E. E., *A.I.Ch.E. Journal*, 4, 343 (1958).

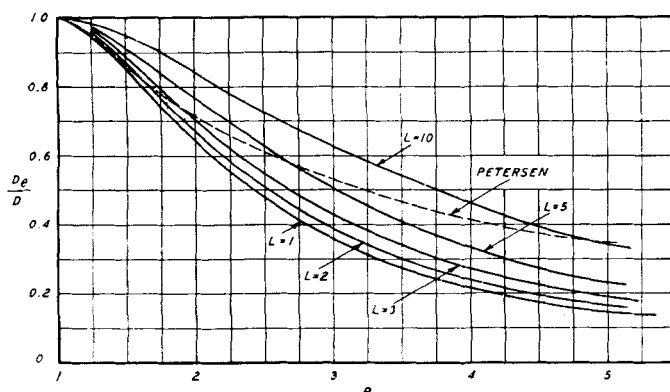


Fig. 1.

where $L = 2R$, the values are virtually identical with Petersen's.

Equation (5) has certain unusual properties which merit comment. The function $L/(L+1)^2$ is relatively insensitive to the value of L , varying only about threefold as L increases from 1 to 10; this means that the convergence factor is much more sensitive to the degree of change of pore cross section (or R) than it is to the relative lengths of the bulges or constrictions. Also $L/(1+L)^2$ is a maximum at $L = 1$ and has the same value for $1/L$ as it does for L ; that is to say, the convergence factor will be the same whether the pore con-

strictions occupy a given fraction of the pore length of the pore bulges occupy that fraction, and it will be smallest when the bulges and constrictions occupy equal fractions of the pore length.

Failure to consider entrance and exit effects in the preceding derivation amounts to assuming that all flux lines are parallel to the pore walls at all points. Clearly however there will be parts of the pore where this is not the case, and thus a certain fraction of the pore volume will not contribute to the flux. Consequently the convergence factor calculated from Equation (5) will be greater than the true convergence factor for the corres-

ponding irregular pore, or, conversely, pore dimensional ratios calculated from a measured convergence factor via Equation (5) will always be larger than the actual values. It should however be pointed out that the actual diffusional flow process through a porous solid is an impressively complex matter involving convergence effects, flow splitting, and multidirectional flow paths, of which the individual contributions to the flow resistance are unknown. One may lump all these effects into either a tortuosity factor or a convergence factor; both are calculational conveniences, but in the absence of a detailed knowledge of the geometry of a porous solid, neither throws any direct light on the flow mechanism. In view, therefore, of the limitations attendant upon employing any parallel capillary model to describe a random porous medium, further refinement of the irregular pore model hardly seems profitable.

NOTATION

- c_i = constant upstream concentration
- c_0 = constant downstream concentration
- D = diffusion coefficient
- D_e = effective diffusivity
- L = ratio of length of pore bulges to pore constrictions
- R = ratio of maximum to minimum pore diameter

Slot Capacity of Bubble Caps

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In setting up their differential equations Rogers and Thiele (1) overlooked the directional property of the differential slot height and did not put in a necessary minus sign. Later they reversed the integration limits without changing the sign. The final equation for rectangular slots happens to be correct, because the slot width does not vary with the slot height, and two negatives make a positive. However their equations for trapezoidal and triangular slots are incorrect; the capacities of trapezoidal and tri-

angular slots are underestimated by 14.3 and 50%, respectively.

DERIVATION OF EQUATION

For the convenience of comparison the notation used here is the same as that used by Bolles (2), except that the slot-height variable and the slot opening start from the top of a slot, as illustrated in Figure 1.

Trapezoidal slots

Application of the orifice equation to a

differential element of slot area gives the vapor velocity through the differential area

$$u = K_s \sqrt{2g \left(\frac{\rho_L - \rho_v}{\rho_v} \right) \frac{h}{12}} \quad (1)$$

The vapor flow rate through the differential area is given by

$$dV_s = u dA$$

$$= K_s \sqrt{2g \left(\frac{\rho_L - \rho_v}{\rho_v} \right) \frac{h}{12}} \frac{w dh}{144} \quad (2)$$